

Dynamical behavior of the entanglement, purity and energy between atomic qubits in motion under the influence of thermal environment

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Abstract. The entanglement, purity and energy of two isolated two-level atoms which are initially prepared in Bell state and each interacts with a thermal cavity field are investigated by considering the atomic motion and the field-mode structure. We achieve the analytical solutions of the atomic qubits by using the algebraic dynamical approach and the influences of the field-mode structure parameter, the strength of the thermal field and the detuning on the entanglement, purity and energy are discussed. We also investigate the state evolution of the atomic qubits based on the entanglement-purity-energy diagrams. Our results show that the disentanglement process of the atomic qubits accompanies by excitations transferring from atomic subsystem to cavity field modes and atomic state from a pure state convert to the mixed states.

1. Introduction

Entanglement is one of the most remarkable features of quantum mechanics and has many practical applications in quantum information processing [1]. However, realistic quantum systems are inevitably influenced by the surrounding environment, which always leads to decoherence of the quantum states. Particularly, the thermal field is frequently discussed in this problem. A thermal field, which is emitted by a source in thermal equilibrium at temperature T , is a highly chaotic field with minimal information about its mean value of the energy. However, such a chaotic field can entangle qubits that are prepared initially in a separable state [2], lead to entangled states in the interaction of a single qubit in a pure state with a thermal field regardless of the temperature of the field and reduce the system to a mixed state when the field variables are traced over [3]. The influence of the thermal field strength on the atom-atom entanglement [4] and atom-field entanglement [5] have also been investigated. Besides, Zheng [6] proposed a scheme for realizing two-qubit quantum phase gates with atoms in a thermal cavity. Jin [7] suggested a scheme of teleporting a two-atom entangled state with a thermal cavity and the success probability can reach 1.0.

In this paper, we consider two isolated two-level atoms each interacting with a single-mode thermal cavity field. The effects of the atomic motion and the field-mode structure are considered at the same time. The atomic motion and the field-mode structure not only lead to nonlinear transient effects in the atomic population [8, 9], which are similar to self-induced transparency and adiabatic effects, but also give rise to the periodic evolution of the entropy squeezing [10], the field entropy, the atomic inversion [11] and the entanglement [4, 5]. Other effects by regulating the field-mode structure parameters have also been observed, e.g. decreasing the squeezing in the two-photon JC model [12], operating the entanglement and realizing the quantum gate operation [13]. The recent cavity quantum electrodynamics experiments which use an atomic beam passing along the axis of a rectangular or cylindrical cavity provide the feasibility of discussing the different field mode structures [14, 15]. Then, in the present work, we are interested in the effects of the atomic motion and the field-mode structure on the time evolution of the atomic state in thermal cavity field environment. We suppose the two atoms are initially prepared in one of the Bell states, which is a maximal entangled pure state. The previous studies of maximally entangled states usually use entanglement, purity and energy to characterized the set of two-qubit states [16, 17, 18, 19]. As the atom-field interacting process accompanied by exchanging excitations between atoms and fields, the energy in the system is a direct influences on the entanglement and mixedness properties. On the other hand, the initial pure state of the atomic qubits must be influenced by interacting with a thermal cavity environment, so the mixedness is an important characteristic in the process of entanglement evolution. As a result, we will investigate the time evolution of the entanglement, purity and energy via manipulating of the field-mode structure parameters, the strength of the thermal field and the detunings between atoms and thermal cavity fields. The relationships among entanglement, pure and energy will also be presented with entanglement-pure-energy (EPE) diagram, which can offer a nice visual to the allowed state of the atomic qubits.

The paper is organized as follows: In Sec.2, we first describe the model under consideration and then derive the exact expression for the atomic reduced density matrix using algebraic dynamical approach [20, 21, 22, 23]. The quantities used to quantify the entanglement, purity and energy of atomic qubits are also defined in this section. Sec.3 is devoted to investigate the time evolution of entanglement, purity and energy for the atomic qubits. In Sec.4, we discuss the time evolution of the atomic qubits with a EPE diagram. Finally, we present our conclusion in Sec.5.

2. Model

We consider two identical moving two-level atoms (A and B) and two spatially separated cavities (a and b) with non-decaying single mode fields by using very high quality factor cavities [14, 24]. Atoms A and B fly through cavities a and b with a constant velocity, respectively. We suppose the two subsystems Aa and Bb are identical with same value

of atom-field coupling strength, frequencies and field-mode structure. The Hamiltonian for the considered system in the rotating-wave approximation can be written as ($\hbar = 1$)

$$\begin{aligned} H &= H_1 + H_2, \\ H_1 &= \omega_c a^\dagger a + \omega_0 S_z^A + g f(z)(a^\dagger S_-^A + a S_+^A), \\ H_2 &= \omega_c b^\dagger b + \omega_0 S_z^B + g f(z)(b^\dagger S_-^B + b S_+^B) \end{aligned} \quad (1)$$

where H_1 and H_2 are the Hamiltonians for subsystems Aa and Bb, respectively. a^\dagger and a (b^\dagger and b) are the creation and annihilation operators of the cavity field a (b). S_+^i , S_-^i and S_z^i represent the atomic raising, lowering and inversion operators of the atom i ($i = A, B$). ω_c and ω_0 are the frequencies for the field a and the atom A (or the field b and the atom B), respectively. g is the atom-field coupling strength, and $f(z)$ is the shape function of the cavity field mode. When the interaction energy of atom-field coupling is much larger than the transverse kinetic energy spread of the atom, we can neglect the transverse velocity spread and restrict our investigation to atomic motion along the cavity axis (z axis). Then the atomic motion is incorporated into $f(z)$ as follows

$$f(z) \rightarrow f(vt), \quad (2)$$

where v is the atomic motion velocity. In this regard the cavity field-mode TEM_{mnp} is defined like $f(vt) = \sin(p\pi vt/L)$, where p represents the number of half wavelengths of the field-mode inside a cavity with length L . If the atom passes through the cavity so fast that the atomic motion can be considered as a constant. For a proper choice of the atomic motion velocity $v = gL/\pi$, then $\int_0^t f(vt') dt' = [1 - \cos(pgt)]/pg$.

In the following, we propose the algebraic dynamical approach to derive the time evolution operator and the density operator based on the Hamiltonian (1). The key idea of the algebraic dynamical approach is introducing a canonical transformation operator that transforms the Hamiltonian into a liner function in terms of a set of Lie algebraic generators. According to algebraic dynamics, linear systems are integrable and solvable, then the time evolution operator and the density operator can be obtained easily. In the case of symmetric atom-field interaction, the two subsystems are completely equivalent. For simplicity, we will work with the subsystem Aa. A straightforward analysis of the Hamiltonian (1) shows that the total excitation number for subsystem Aa is

$$N_1 = a^\dagger a + S_z^A + \frac{1}{2}, \quad (3)$$

which is a conserved quantity for the subsystem Aa and commutes with the Hamiltonian H_1 . Based on the algebraic dynamical approach, introducing SU(2) algebra generators $\{J_0, J_+, J_-\}$, with $J_0 = S_z^A$, $J_+ = N_1^{-1/2} a S_+^A$, $J_- = N_1^{-1/2} a^\dagger S_-^A$, which are nonlinear expressions and obey the following commutation relations

$$[J_0, J_+] = J_+, \quad [J_0, J_-] = -J_-, \quad [J_+, J_-] = 2J_0. \quad (4)$$

In terms of the SU(2) algebra generators and the canonical transformation operator $U_g = \exp(\theta J_+ - \theta J_-)$, we can obtained the time evolution operator of the subsystem Aa

$$U_1(t) = e^{-iH_1 t} = U_g e^{-i(U_g^{-1} H_1 U_g) t} U_g^{-1}$$

$$= e^{-iE_1 t} \left[\cos \frac{\lambda t}{2} - 2iJ_0 \sin \frac{\lambda t}{2} \cos 2\theta + i(J_+ + J_-) \sin \frac{\lambda t}{2} \sin 2\theta \right]. \quad (5)$$

where $E_1 = \omega_c(N_1 - \frac{1}{2})$, $\theta = -\arctan[(\sqrt{\Delta^2/4 + g'^2 N_1} - \Delta/2)/g'N_1^{1/2}]$, $\lambda = \sqrt{\Delta^2 + 4g'^2 N_1}$, $g' = g\alpha/t$, $\alpha = \int_0^t f(vt')dt' = [1 - \cos(pgt)]/pg$ and $\Delta = \omega_0 - \omega_c$ is the detuning between the atom A and the cavity a .

What should be noticed here is that using canonical transformation operator to diagonalize the nonlinear Hamiltonian (1) doesn't change its intrinsic qualities. Likewise, we can get the evolution operator U_2 of the subsystem Bb , which has the similar form as U_1 .

$$U_2(t) = e^{-iE_2 t} \left[\cos \frac{\eta t}{2} - 2iL_0 \sin \frac{\eta t}{2} \cos 2\phi + i(L_+ + L_-) \sin \frac{\eta t}{2} \sin 2\phi \right]. \quad (6)$$

where $E_2 = \omega_c(N_2 - \frac{1}{2})$, $\phi = -\arctan[(\sqrt{\Delta^2/4 + g'^2 N_2} - \Delta/2)/g'N_2^{1/2}]$, and $\eta = \sqrt{\Delta^2 + 4g'^2 N_2}$. $N_2 = b^\dagger b + S_z^B + \frac{1}{2}$ is the total excitation number for subsystem Bb . $\{L_0, L_+, L_-\}$ are the $SU(2)$ algebra generators with $L_0 = S_z^B$, $L_+ = N_2^{-1/2}bS_+^B$, $L_- = N_2^{-1/2}b^\dagger S_-^B$.

Throughout this paper we suppose the two atoms AB to be initially prepared in one of the Bell states, $|\Psi\rangle = \frac{1}{\sqrt{2}}(|eg\rangle + |ge\rangle)$, and the two thermal cavity fields ab are in single-mode thermal field states $\rho_a(0) = \sum_{n=0}^{\infty} P_n |n\rangle\langle n|$, $\rho_b(0) = \sum_{m=0}^{\infty} P_m |m\rangle\langle m|$. As a result, the initial density operators for the two atoms and the two thermal cavity fields can be written as

$$\begin{aligned} \rho_{AB}(0) &= |\Psi\rangle\langle\Psi| = \frac{1}{2}(|eg\rangle\langle eg| + |eg\rangle\langle ge| + |ge\rangle\langle eg| + |ge\rangle\langle ge|), \\ \rho_f(0) &= \rho_a(0) \otimes \rho_b(0) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} P_n P_m |nm\rangle\langle nm|, \end{aligned} \quad (7)$$

where $P_n = \frac{k^n}{(k+1)^{n+1}}$, $P_m = \frac{l^m}{(l+1)^{m+1}}$. $k = 1/[\exp(\omega_c/T_a) - 1]$ and $l = 1/[\exp(\omega_c/T_b) - 1]$, k and l are the mean photon numbers of the thermal cavity field mode a and the thermal cavity field mode b , corresponding to the temperatures T_a and T_b , respectively.

Then, the initial density operator for the total system can be derived as

$$\begin{aligned} \rho_{AB-f}(0) &= \rho_{AB}(0) \otimes \rho_f(0) \\ &= \frac{1}{2} \sum_n \sum_m P_n P_m (|engm\rangle\langle engm| + |engm\rangle\langle gnem| \\ &\quad + |gnem\rangle\langle engm| + |gnem\rangle\langle gnem|), \end{aligned} \quad (8)$$

where the $|engm\rangle$ indicates that atom A is in the excited state and atom B is in the ground state, field mode A and field mode B are in the states $|n\rangle$ and $|m\rangle$, respectively.

The initial state (8) under the action of the operator $U_1(t) \otimes U_2(t)$ evolves to

$$\rho_{AB-f}(t) = U_1(t)U_2(t)\rho_{AB-f}(0)U_2^\dagger(t)U_1^\dagger(t). \quad (9)$$

Then, from Eq.(9), we can get the reduced density matrix $\rho_{AB}(t)$ of the subsystem AB by tracing over the thermal cavity field variables. In terms of the atomic basis states

$|gg\rangle$, $|ge\rangle$, $|eg\rangle$, and $|ee\rangle$, the reduced density operator $\rho_{AB}(t)$ can be expressed as

$$\rho_{AB}(t) = \text{Tr}_f[\rho_{AB-f}(t)] = \begin{pmatrix} x_1 & 0 & 0 & 0 \\ 0 & x_2 & x_3 & 0 \\ 0 & x_4 & x_5 & 0 \\ 0 & 0 & 0 & x_6 \end{pmatrix} \quad (10)$$

where $x_1 + x_2 + x_5 + x_6 = 1$,

$$\begin{aligned} x_1 &= \frac{1}{2} \sum_n \sum_m \{ P_{n-1} P_m [\sin^2(\frac{\lambda_n t}{2}) \sin^2(2\theta_n)] [\cos^2(\frac{\eta_m t}{2}) + \sin^2(\frac{\eta_m t}{2}) \cos^2(2\phi_m)] \\ &\quad + P_n P_{m-1} [\cos^2(\frac{\lambda_n t}{2}) + \sin^2(\frac{\lambda_n t}{2}) \cos^2(2\theta_n)] [\sin^2(\frac{\eta_m t}{2}) \sin^2(2\phi_m)] \}, \\ x_2 &= \frac{1}{2} \sum_n \sum_m \{ P_n P_m [\cos^2(\frac{\lambda_n t}{2}) + \sin^2(\frac{\lambda_n t}{2}) \cos^2(2\theta_n)] \\ &\quad \times [\cos^2(\frac{\eta_{m+1} t}{2}) + \sin^2(\frac{\eta_{m+1} t}{2}) \cos^2(2\phi_{m+1})] \\ &\quad + P_{n-1} P_{m+1} [\sin^2(\frac{\lambda_n t}{2}) \sin^2(2\theta_n)] [\sin^2(\frac{\eta_{m+1} t}{2}) \sin^2(2\phi_{m+1})] \}, \\ x_3 &= \frac{1}{2} \sum_n \sum_m \{ P_n P_m [\cos(\frac{\lambda_n t}{2}) + i \sin(\frac{\lambda_n t}{2}) \cos 2\theta_n] \\ &\quad \times [\cos(\frac{\lambda_{n+1} t}{2}) + i \sin(\frac{\lambda_{n+1} t}{2}) \cos(2\theta_{n+1})] \\ &\quad \times [\cos(\frac{\eta_{m+1} t}{2}) - i \sin(\frac{\eta_{m+1} t}{2}) \cos 2\phi_{m+1}] [\cos(\frac{\eta_m t}{2}) - i \sin(\frac{\eta_m t}{2}) \cos(2\phi_m)] \}, \\ x_4 &= x_3^*, \\ x_5 &= \frac{1}{2} \sum_n \sum_m \{ P_{n+1} P_{m-1} [\sin^2(\frac{\lambda_{n+1} t}{2}) \sin^2(2\theta_{n+1})] [\sin^2(\frac{\eta_m t}{2}) \sin^2(2\phi_m)] \\ &\quad + P_n P_m [\cos^2(\frac{\lambda_{n+1} t}{2}) + \sin^2(\frac{\lambda_{n+1} t}{2}) \cos^2(2\phi_{n+1})] \\ &\quad \times [\cos^2(\frac{\eta_m t}{2}) + \sin^2(\frac{\eta_m t}{2}) \cos^2(2\phi_m)] \}, \\ x_6 &= \frac{1}{2} \sum_n \sum_m \{ P_n P_{m+1} [\cos^2(\frac{\lambda_{n+1} t}{2}) + \sin^2(\frac{\lambda_{n+1} t}{2}) \cos^2(2\theta_{n+1})] \\ &\quad \times [\sin^2(\frac{\eta_{m+1} t}{2}) \sin^2(2\phi_{m+1})] \\ &\quad + P_{n+1} P_m [\sin^2(\frac{\lambda_{n+1} t}{2}) \sin^2(2\theta_{n+1})] [\cos^2(\frac{\eta_{m+1} t}{2}) + \sin^2(\frac{\eta_{m+1} t}{2}) \cos^2(2\phi_{m+1})] \}, \end{aligned} \quad (11)$$

and $\lambda_n = \sqrt{\Delta^2 + 4g'^2 n}$, $\eta_m = \sqrt{\Delta^2 + 4g'^2 m}$, $\theta_n = -\arctan[(\sqrt{\Delta^2/4 + g'^2 n} - \Delta/2)/g'n^{1/2}]$, $\phi_m = -\arctan[(\sqrt{\Delta^2/4 + g'^2 m} - \Delta/2)/g'm^{1/2}]$.

Based on the analytical solution of $\rho_{AB}(t)$, we can conveniently do some approximation in analyzing the numerical results in the following section. Besides, we can define the quantities to quantify the entanglement, purity and energy of the atomic qubits and give their expressions in terms of the matrix elements of $\rho_{AB}(t)$.

We adopt Wootters' concurrence as a measure of entanglement in this discussion [25], which is denoted as $C_{AB} = \text{Max}\{0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}\}$, and λ_i are the eigenvalues of the matrix $(\rho_{AB}\tilde{\rho}_{AB})$ in non increasing order. Following Eq.(10), the expression of C_{AB} turns out to be

$$C_{AB} = 2\text{Max}\{0, |x_3| - \sqrt{x_1 \times x_6}\}. \quad (12)$$

When the value of C_{AB} is positive, the atomic system is entangled. $C_{AB} = 1$ corresponds to the maximal entanglement state, while $C_{AB} = 0$ indicates the atom A and the atom B are separable.

The energy U_{AB} of subsystem AB is defined here as the expectation value of the Hamiltonian $H_{AB} = \omega_0 S_z^A + \omega_0 S_z^B$. We set $\omega_0 = 1$, then the energy U_{AB} can be obtained based on the expression of Eq.(10)

$$U_{AB} = \text{Tr}\{\rho_{AB-f}(t)H_{AB}\} = x_6 - x_1 \quad (13)$$

For two two-level atoms, U_{AB} ranges from -1 for $\rho_{AB} = |gg\rangle\langle gg|$ to 1 for $\rho_{AB} = |ee\rangle\langle ee|$.

To quantify the mixedness of the state $\rho_{AB}(t)$, we use the purity

$$P_{AB} = \text{Tr}\{\rho_{AB}^2(t)\} = x_1^2 + x_2^2 + x_5^2 + x_6^2 + 2x_3x_4 \quad (14)$$

For the atomic qubits, P_{AB} ranges from $1/d$ for completely mixed state to 1 for pure state for d -dimensional systems, which is closely to the linear entropy measure of mixedness.

The aim of this paper is to address the question how the atomic motion and the field-mode structure influence the state $\rho_{AB}(t)$ of the atomic qubits in the cases of thermal environment. We know that the two atoms are initially in the maximal entanglement state, and the atomic state will evolves with time followed by the variation of the entanglement, purity and the transfer of the energy. Their time evolution will be discussed in the next section.

3. Entanglement, purity and energy versus time

There are three controllable parameters in the analytical expression of $\rho_{AB}(t)$: the field-mode structure parameter, the mean photon number in each cavity and the detuning. In this section, we will discuss their effects of the three parameters on the time evolution of the entanglement, purity and energy of the atomic subsystem.

In Fig.1 and Fig.2 we plot the time evolution of C_{AB} , P_{AB} and U_{AB} affected by different values of the field-mode structure parameters and the mean photon number in the situation of exact resonance. Form Eq.(11), we can easily find that for resonant atom-field coupling, $\Delta=0$, $\lambda_n = 2g'\sqrt{n}$, $\eta_m = 2g'\sqrt{m}$, $\sin 2\theta_n = \sin 2\phi_m = -1$, $\cos 2\theta_n = \cos 2\phi_m = 0$. The elements of the matrix ρ_{AB} expressed in Eq.(11) convert to

$$x_1 = \frac{1}{2} \sum_n \sum_m [P_{n-1}P_m \sin^2(g't\sqrt{n}) \cos^2(g't\sqrt{m})]$$

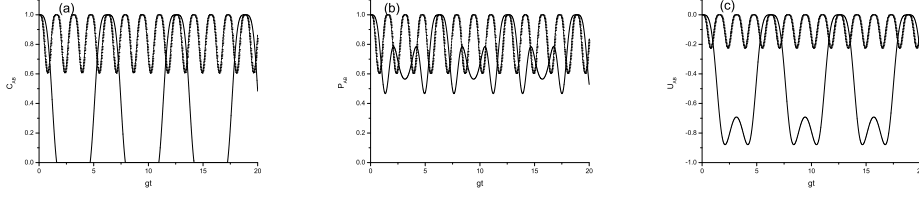


Figure 1. The evolution of C_{AB} , P_{AB} and U_{AB} for different field-mode structure parameters, $p = 1$ (solid) and $p = 4$ (dotted). The two subsystems are symmetric for $k = l = 0.1$ and the atom and the field are in exact resonance. (a) C_{AB} versus time gt ; (b) P_{AB} versus time gt ; (c) U_{AB} versus time gt .

$$\begin{aligned}
& + P_n P_{m-1} \cos^2(g't\sqrt{n}) \sin^2(g't\sqrt{m})], \\
x_2 = & \frac{1}{2} \sum_n \sum_m [P_n P_m \cos^2(g't\sqrt{n}) \cos^2(g't\sqrt{m+1}) \\
& + P_{n-1} P_{m+1} \sin^2(g't\sqrt{n}) \sin^2(g't\sqrt{m+1})], \\
x_3 = & \frac{1}{2} \sum_n \sum_m [P_n P_m \cos(g't\sqrt{n}) \cos(g't\sqrt{n+1}) \cos(g't\sqrt{m}) \cos(g't\sqrt{m+1})], \\
x_4 = & x_3, \\
x_5 = & \frac{1}{2} \sum_n \sum_m [P_{n+1} P_{m-1} \sin^2(g't\sqrt{n+1}) \sin^2(g't\sqrt{m}) \\
& + P_n P_m \cos^2(g't\sqrt{n+1}) \cos^2(g't\sqrt{m})], \\
x_6 = & \frac{1}{2} \sum_n \sum_m [P_n P_{m+1} \cos^2(g't\sqrt{n+1}) \sin^2(g't\sqrt{m+1}) \\
& + P_{n+1} P_m \sin^2(g't\sqrt{n+1}) \cos^2(g't\sqrt{m+1})]. \tag{15}
\end{aligned}$$

3.1. the effects of field-mode structure parameters

The atomic initial state is a Bell state, which is a pure state with $P_{AB} = 1$ as well as a maximal entanglement state with $C_{AB} = 1$. However, the initial energy for the atomic subsystem is zero, that is $U_{AB} = 0$. Fig.1 illustrates the cases when the atom is in motion at the velocity $v = gL/\pi$ for parameters $p = 1$ and $p = 4$, respectively. It has been studied that when the atomic motion is considered, the time behaviors of field entropy, atomic inversion [11], and entropy squeezing [10] are periodical, and their evolution periods are shortened with the increase of parameter p . Similar behaviors occur in this work. From Fig.1 we can find that the evolution periods are decreased with the increase of parameter p . This is because the time factor is the scaled time gt when the atomic motion is neglected, and is $g't$ when the atomic motion is taken into account. $g't = [1 - \cos(pgt)]/p$ is a periodical function on the scaled time gt with period $2\pi/p$. In addition, the amplitudes for C_{AB} , P_{AB} and U_{AB} are reduced while their

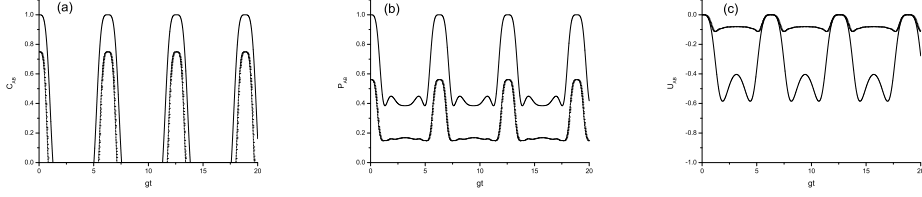


Figure 2. The evolution of C_{AB} , P_{AB} and U_{AB} for different thermal field strengths, $k = l = 0.5$ (solid), $k = l = 5$ (dotted). The atom and the field are in exact resonance and the field-mode structure parameter $p = 1$. (a) C_{AB} versus time gt ; (b) P_{AB} versus time gt ; (c) U_{AB} versus time gt .

maximum values are still unchanged. That is, compared with $p = 1$, the “sudden death of entanglement” disappears, the maximum mixedness of the atomic state reduces and the energy exchange between atoms and field modes decreases with the increase of the field-mode structure parameter p .

3.2. the effects of mean photon number in each cavity

What we talked about in Fig.1 is just limited to the situation of very weak thermal field with mean photon number $k = l = 0.1$ in each cavities. Then we are interested in how the atomic qubits evolves as the mean photon number in each cavity increases. It has been demonstrated that thermal cavity field can lead to entangled state of quantum qubits interacting with it [2, 3], while strong thermal cavity field can inhibit the atom-atom entanglement [4] and atom-field entanglement [5]. Here, we pay our attention to the influence of mean photon number on the non-local atom-atom entanglement, in the case of exact resonance, as is shown in Fig.2(a). In addition, in Fig.2(b) and Fig.2(c), further study is employed to the time evolution of purity and energy, which can help us to get more information about entanglement evolution. From Fig.2 we can find that with the increase of mean photon number in each cavity, both the amplitudes and the maximum values for C_{AB} , P_{AB} and U_{AB} decrease. That is, compared with the case of weak thermal cavity field, the atomic qubits, which couples to strong thermal cavity fields, can not evolve to a maximal entanglement state with $C_{AB} = 1$ as well as a pure state with $P_{AB} = 1$. Meanwhile, the time interval of the “sudden death of entanglement” lengthens, the maximum mixedness increases, while the energy transfer between atomic qubits and field modes in each interacting period is more and more less.

3.3. the effects of detunings between atom and field

The effects of detuning on the entanglement, purity and energy between atom A and atom B are depicted in Fig.3. In Fig.3(a), C_{AB} oscillates at first and as time evolves it will maintain the maximal entanglement state when the influence of the atomic motion is considered and $\Delta \neq 0$. Moreover, the larger the value of detuning, the faster the

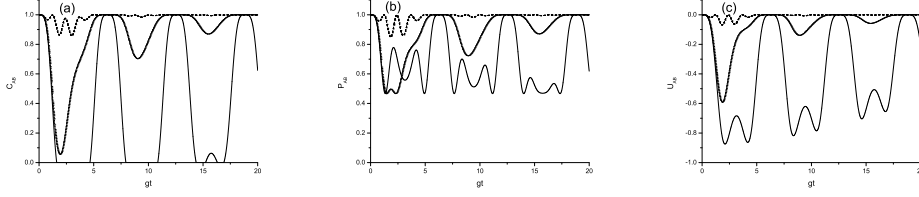


Figure 3. The evolution of C_{AB} , P_{AB} and U_{AB} for different detuning, $\Delta = 0.1$ (solid), $\Delta = 1$ (dotted) and $\Delta = 5$ (dashed). The two subsystems are symmetric for $k = l = 0.1$ and the field-mode structure parameter $p = 1$. (a) C_{AB} versus time gt ; (b) P_{AB} versus time gt ; (c) U_{AB} versus time gt .

C_{AB} reaches the stable maximal value 1. When $\Delta \gg g$, the oscillating almost vanishes and the two atoms nearly entangle maximally all the time as time evolves. Similar behavior to the time evolution of P_{AB} is depicted in Fig.3(b). In Fig.3(c) we can find that with the increase of Δ , the amplitude of U_{AB} is reduced gradually, and large values of Δ make it more easy to get to zero. In a word, with the increase of detuning, the atomic subsystem is almost "frozen" in the initial state. This can be explained as follows: on the one hand, based on the expression of ρ_{AB} , for weak thermal cavity fields $k = l = 0.1$ and large detuning $\Delta \gg g$, $\lambda_n = \eta_m \approx \Delta$, $\sin 2\theta_n = \sin 2\phi_m \approx 0$, $\cos 2\theta_n = \cos 2\phi_m \approx 1$. As a result, $x_1 = x_6 \approx 0$, $x_2 = x_3 = x_4 = x_5 \approx \frac{1}{2} \sum_n \sum_m P_n P_m$. Then, $C_{AB} = 2 \text{Max}\{0, |x_3| - \sqrt{x_1} \times x_6\} \approx \sum_n \sum_m P_n P_m = 1$, $P_{AB} = \text{Tr}\{\rho_{AB}^2(t)\} = x_1^2 + x_2^2 + x_5^2 + x_6^2 + 2x_3x_4 \approx \sum_n \sum_m P_n P_m = 1$, $U_{AB} = \text{Tr}\{\rho_{AB-f}(t)H_{AB}\} = x_6 - x_1 \approx 0$; on the other hand, the interacting process between atoms and fields is accompanied by the transfer of the excitation between the localized atom and cavity mode, which depends on the atom-field coupling and is distinctly influenced by the value of detuning. Larger detuning can inhibit the atom-field coupling and restrain this transfer process greatly, therefore the initially maximal entanglement pure state can be "frozen" in the atomic subsystem.

4. Entanglement-purity-energy diagram

In this section, we devote to investigate the relationships among entanglement, purity and energy for atomic subsystem, which reflects much of the nontrivial information about the particular atomic state in the atom-field interacting process. Here, we limit our study to the weak thermal field with $k = l = 0.1$ and the field-mode structure parameter $p = 1$ in resonant situation. Under these conditions, $\cos 2\theta_n = \cos 2\phi_m = 0$, $\sin 2\theta_n = \sin 2\phi_m = -1$. We plot entanglement-purity-energy diagram in Fig.4(a) and show its projections on entanglement-energy and entanglement-purity planes in Fig.4(b) and Fig.4(c), respectively. At initial time, $C_{AB} = 1$, $P_{AB} = 1$ and $U_{AB} = 0$, the atomic qubits in the maximal entangled state. From Fig.4(b) we can find that the disentanglement process accompanies by excitations transferring from atomic subsystem

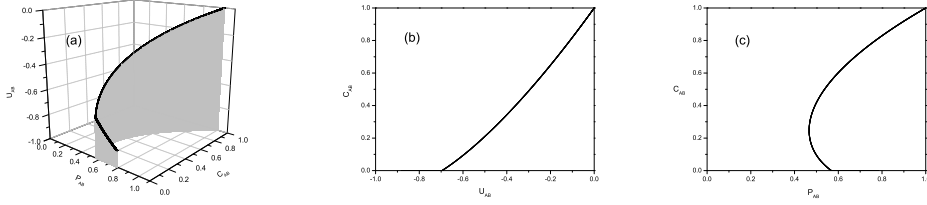


Figure 4. The relationship between entanglement, mixedness and energy for $p = 1$, $\Delta = 0$, $k = l = 0.1$. (a) three-dimensional diagram for C_{AB} , P_{AB} and U_{AB} ; (b) C_{AB} versus U_{AB} ; (c) P_{AB} versus U_{AB} .

to cavity field modes and atomic state from a pure state convert to mixed states. The minimal energy for atomic subsystem is about -0.7 when the two atoms are separable, and the maximum value is zero when the two atoms are in the maximal entanglement state. This suggests the atomic state can not evolve to $\rho_{AB} = |gg\rangle\langle gg|$ ($U_{AB} = -1$) or $\rho_{AB} = |ee\rangle\langle ee|$ ($U_{AB} = 1$) in the atom-field interaction process, which can also be confirmed in Fig.4(c). When $C_{AB} = 0$, $P_{AB} \neq 1$, the atomic qubits is in mixed state when they are separable. While $C_{AB} = 1$ corresponding to $P_{AB} = 1$, this indicates that the atomic qubits can just realize the maximal entanglement pure state but the maximal entanglement mixed state can not be obtained.

5. Conclusion

In this paper, we employed three parameters—entanglement, purity and energy to describe the information about two distant atoms which are initially prepared in Bell state. Our results showed that considering the atomic motion and the field-mode structure can lead to the periodic evolution of entanglement, purity and energy. With the increase of field-mode structure parameter p , both their evolution periods and their amplitudes are decreased while their maximum values are unchanged. However, strong thermal field can reduce the peak values of entanglement, purity and energy of the atomic qubits and make the atomic state initially in a pure state to mixed states. Meanwhile, in such a chaotic field, energy transfer between atoms and fields is more and more less with the increase of thermal field strength. In addition, large detuning is in favour of reducing their oscillation time and "freezing" the initial maximal entanglement state in the atomic subsystem. We also analyzed the possible state that the atomic qubits may evolves into. From the entanglement-purity-entanglement diagram we found that the disentanglement process for the atomic subsystem accompanies both by the excitation transferring from atomic subsystem to cavity field modes and the state converts from a pure state to mixed states. Our number results showed that, when the atomic state is in the maximal entanglement state, it is in a pure state at the same time; when the two atoms are separable, the atomic state is in a mixed state. However, in the atom-field interacting process, the state for atomic qubits can not evolve to the maximal

entanglement mixed state.

Acknowledgment

This research is supported by the National Natural Science Foundation of China under Grant No. 10704031, the National Science Foundation for Fostering Talents in Basic Research of the National Natural Science Foundation of China Under Grant No. J0630313, the fundamental Research Fund for Physical and Mathematical of Lanzhou University Under Grant No. Lzu05001, and the Natural Science Foundation of Gansu Under Grant No. 3ZS061-A25-035.

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